**CFD 502 - Project 2**

**Steady state of unconfined flow in shallow aquifer on inclined base**

**Theory**

Figure 1 is a profile view of unconfined saturated groundwater flow through a soil layer or aquifer generated by uniform recharge in space and time at the rate *r* referenced to a unit horizontal area. The aquifer is schematised as underlain by an impervious, infinitely wide plane (2-D) of length ***L*** that isinclined against the horizontal at the angle ***ϕ***, and is assumed homogeneous, with constant hydraulic conductivity ***k*** and drainable porosity (specific yield) ***n***. We apply the Dupuit-Forchheimer theory, considering the flow as shallow (*h* << *L*), with negligible curvature of the streamlines, so that the pressure in the water column of height ***h*** normal to the bed may be taken as hydrostatic.



**Fig. 1** Cross-sectional schematic diagram of sloping soil layer, with definition of symbols.

Under these assumptions and using the notation shown in Fig. 1 and explained above, the discharge ***q*** per unit aquifer width (law of Darcy), at time ***t*** and location ***x*** measured from the top of the hill along the inclined base (orientation that makes the discharge positive in the +*x-*direction), is formulated as

. (1)

The differential storage balance equation is

. (2)

The right-hand side of eq. 2 represents the flux falling on an element of length Δ*x* (Wooding and Chapman, 1966; it derives from the scalar product of the recharge vector and the unit normal of a free-surface element (Akylas et al., 2006).The term *r*(∂*h*/∂*x*)sin*ϕ* is rarely included in the volume balance and may be safely neglected relative to *r*cos*ϕ* when *r*/*k* << 1 (Chapman, 2005). Substituting *q* from eq. 1 into the simplified eq. 2 yields the nonlinear governing equation derived by Boussinesq:

. (3)

**Dimensionless Formulation**

We may then proceed with dimensionless formulation and introducing into equations (1-3) the variables

 (4)

we conclude with the dimensionless forms

, ,  (5)

where the similarity of the problem is reflected in the value of parameter *m.* The previous equations (5a,b,c) have been studied extensively and there are some solutions available especially for the case with boundary conditions *q*(0) = 0 and *h*(*L*)=0. The specific set of the boundary conditions in dimensionless form read

**Boundary Conditions:** H(1) = 0, Q(0) = 0 . (6)

The boundary condition at *X*=0, should be treated carefully since it can be proven that for values of parameter *m* > 2, leads to H(0) = 0, while for values of *m* < 2 leads to *H*’(0)=*m*.

**References**

Akylas, E., Koussis, A.D., and A.N. Yannacopoulos, 2006. Analytical solution of transient flow in a sloping soil layer with recharge.*Hydrological sciences journal* 51(4) (2006): 626-641.

Akylas E., Gravanis E., and A.D. Koussis, 2013. Quasi Steady approximation in sloping aquifers. Submitted to WRR.

Chapman, T. G.,1963. Effects of ground-water storage and flow on the water balance. *Water Resources, Use and Management* (1963): 290-301.

Wooding, R.A., Chapman, T.G., 1966. Groundwater ﬂow over a sloping impermeable layer, 1. Application of the Dupuit– Forchheimer assumption. J. Geophys. Res. 71, 2895– 2902.

**Questions**

Write a short study facing the following tasks:

1. Calculate analytically the steady-state solution for the water profile H(X) for a horizontal aquifer and integrate in order to calculate the total water storage *S =* 0∫*1HdX* in the aquifer.
2. Write and explain the coding procedure in order to solve the steady-state solution for the profile of *H*(*X*) at different *m* values.
3. Solve and examine the behaviour of the solution at large values of the parameter *m*>2 and low values of *m*<2.
4. Integrate numerically the *H*(*X*) solutions in order to calculate the total water storage *S =* 0∫*1HdX* in the aquifer, as a function of the parameter *m*, and compare the results with the following analytical solution (Akylas et al., 2013):

, where  for , and =1 for .