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Ground temperature estimations using simplified analytical and semi-empirical approaches

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Abstract

In this work, subsurface ground temperature profiles are estimated by exploiting two different approaches. In the first one, an analytical model is examined which, considering a quasi steady state system, implements the superposition of annual and daily sinusoidal fluctuations. In the second one, semi-empirical models are developed based on the general formula of the preceding, by replacing the steady state soil temperature with easily obtained daily average temperatures. Various subsets of soil temperature were used for model development, in order to explore the possibility of minimizing data requirements. Comparison of observational data with model results reveals that the observational patterns of hourly soil temperature are fairly well approximated by both by the analytical and the semi-empirical models. All models seem to capture the main characteristics of the annual course of soil temperature, with the results obtained from the semi-empirical models fluctuating in a much more realistic way than those of the analytical model. It is concluded that the proposed models may serve as useful tools for estimating and predicting soil temperatures to be used as practical reference in various environmental and energy applications.

Keywords: Ground temperature; Temperature distribution; Heat transfer; Heat conduction equation; Modeling; Energy conservation

1. Introduction

The estimation and/or prediction of the subsurface ground temperature profile is commonly required in various environmental and energy applications (e.g., Santamouris, 2007). The measurement of ground temperature profile is, however, not always easy, thus modeling can be a useful tool for providing knowledge of the diurnal and annual variations of the soil temperature at different depths. Modeling approaches include, in general, analytical models (e.g., Moustafa et al., 1981; Swaid and Hoffman, 1989; Smerdon et al., 2006), semi-analytical models (e.g., Yuan et al., 2008), numerical models (e.g., Hanks et al., 1971; van Bavel and Hillel, 1975; Sikora et al., 1990; Mihalakakou et al., 1995; Janssen et al., 2004) improved or modified numerical solutions by three-dimensional thermal response (e.g., Zoras et al., 2001,2002) and empirical models (e.g., Parton and Logan, 1981; Kemp et al., 1992; Kayali et al., 1998; Al-Teneemi and Harris, 2001; Al-Ajmi et al., 2006). In addition, models based on the Fourier technique (e.g., Carson, 1963; Costello and Braud, 1989; Jacovides et al., 1996; Kumar et al., 2007) and on artificial neural networks (e.g., Yang et al., 1997; Mihalakakou, 2002) have also been developed. Despite the availability of such a bundle of modeling approaches,

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the tradeoff between the accuracy of the obtained modeled results, on the one hand, and data availability along with cost and time effectiveness, on the other hand, are the major concerns when selecting a model or a modeling approach.

The purpose of this work is to present simplified modeling tools for soil temperature profiles while keeping the complexity and the data requirement to a minimum. First, a deterministic analytical approach is examined as a means to obtain realistic estimations with minimal requirements, in terms of data, time, and cost with the hope that the more constraining assumptions of an analytical model may be counterbalanced by the easy use and practical importance that it may have in some cases. The model is based on a quasi steady state approach that takes into account the superposition of the annual and daily (diurnal) sinusoidal fluctuation around a constant value of the soil temperature. In addition, semi-empirical models, based on the general formula of the analytical model, are then developed to serve as predictive tools. In the semi-empirical models, the steady state soil temperature is replaced with some easily obtained daily average temperatures, in an attempt to re-introduce a transient forcing term in the model. Model results are then compared to observational data from an experimental plot to evaluate the performance of the developed models. It should also be noted that although the models are expected to be used in the context of various applications, the application described in the present work is limited only to soil depths more related to agricultural problems rather than other environmental applications such as passive heating and cooling of buildings.

2. The analytical model

In the case of an 'infinite depth soil' with an initially uniform temperature $T(z,0) = T_{in}$, which is characterized by the following linearized equation for the temperature diffusion:

$$\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial z^2}, \tag{1}$$

the response to a change of the surface temperature $T(0, t > 0) = T_{in} + \Delta T_0$ is mathematically described by the general solution as (Carslaw and Jaeger, 1986):

$$T(z,t) = T_{in} + \Delta T_0 \cdot \operatorname{erfc}\left(\frac{z}{2\sqrt{D\cdot t}}\right), \tag{2}$$

where t is the time, z the soil depth, D is a representative constant thermal diffusivity and erfc(x) is the error complementary function.

From the previous solution, one can calculate the system response function $u(z,t)$ of the soil to a unit step ΔT_0 change in the surface temperature, $\Delta T_0 = 1$ which occurs at the time limit $\Delta t \to 0$:

$$u(z,t) = \left.\frac{\partial T(z,t)}{\partial t}\right|_{\Delta T_0 \to 1} = -\frac{1}{2t}\cdot \operatorname{erf}\left(\frac{z}{2\sqrt{D\cdot t}}\right). \tag{3}$$

Applying the convolution theorem by integrating properly the product of the system response function (3) with the forcing signal of the surface temperature $\Delta T_0(t)$ (when this signal is known), we conclude the transient solution for the temperature at any time and depth:

$$\begin{aligned} T(z,t) &= T_{in} + \int_0^t \Delta T_0(\tau) u(z, t-\tau)\,d\tau \\ &= T_{in} + \int_0^t \Delta T_0(t-\tau) u(z,\tau)\,d\tau. \end{aligned} \tag{4}$$

In the case where the surface temperature shows a periodic variation, applied for an infinite duration, of the form:

$$T(0,t) = T_a + A_0 \cdot \sin(2\pi f t + \varphi). \tag{5}$$

Hillel (1982) has solved (4) and proved that it results to the quasi-steady state solution:

$$T(z,t) = T_a + A_0 \cdot e^{-z/d} \cdot \sin\left(2\pi f t - \frac{z}{d} + \varphi\right), \tag{6}$$

where T_a is the mean soil temperature, f is the frequency and φ the surface signal phase. The parameter d is the damping depth, which is given by

$$d = \sqrt{\frac{D}{f\pi}}. \tag{7}$$

Note that, since the sinusoidal variation has been applied for an infinite period, there is no transient part in the final solution (6). In other words, the soil at any depth is synchronized to the surface signal, which is why we refer to a quasi-steady state. This is actually not the case when the surface forcing has been applied for short times, where the different depths exhibit a transient, much more complicated behavior. For many systems though, with canonical repeating behavior, the quasi-steady state approximation is reasonably valid.

Based on (6), we extend here the concept of a more realistic physical forcing, where the model would have to account for several signals simultaneously transmitted through the medium. Surface temperature could be then generalized by superposing sinusoidal terms such as

$$T(0,t) = T_a + \sum_{i=1}^{n} A_i \cdot \sin(2\pi f_i t + \varphi_i). \tag{8}$$

Through (6), the quasi-steady soil temperature at any given depth would then be

$$T(z,t) = T_a + \sum_{i=1}^{n} A_i \cdot e^{-z/d_i} \cdot \sin\left(2\pi f_i t - \frac{z}{d_i} + \varphi_i\right). \tag{9}$$

Specifying the surface signal to be a reasonable superposition of two waves, namely the annual period wave ($f = 1/8760\ h^{-1}$) and the respective daily one ($f = 1/24\ h^{-1}$) with variable amplitudes, we approximate the surface temperature as (Garofalakis, 2004):

$$T(0,t) = T_a + A_1 \cdot \sin\left(\frac{2\pi t}{8760} + \varphi_1\right) + A_2 \cdot \left[A_3 - \cos\left(\frac{2\pi t}{8760} + \varphi_2\right)\right] \cdot \sin\left(\frac{2\pi t}{24} + \varphi_3\right). \tag{10}$$

Using the identity $2\cos x \sin y = \sin(y - x) + \sin(x + y)$, after some simple trigonometric calculations, Eq. (10) can be finally written in the general form of (8):

$$\begin{aligned} T(0,t) = {} & T_a + A_1 \cdot \sin\left(\frac{2\pi t}{8760} + \varphi_1\right) + A_2 \cdot A_3 \cdot \sin\left(\frac{2\pi t}{24} + \varphi_3\right) \\ & - \frac{A_2}{2} \cdot \sin\left(\frac{364 \cdot 2\pi t}{8760} - \varphi_2 + \varphi_3\right) - \frac{A_2}{2} \cdot \sin\left(\frac{366 \cdot 2\pi t}{8760} + \varphi_2 + \varphi_3\right). \end{aligned} \tag{11}$$

Therefore, if the surface signal described by (11) is being applied for infinite time, the temperature $T(z,t)$, at the general depth z, is calculated by the direct application of Eq. (9) as follows:

$$\begin{aligned} T(z,t) = {} & T_a + A_1 \cdot e^{-z/d_1} \cdot \sin\left(\frac{2\pi t}{8760} + \varphi_1 - \frac{z}{d_1}\right) + A_2 \cdot A_3 \cdot e^{-z/d_2} \cdot \sin\left(\frac{2\pi t}{24} + \varphi_3 - \frac{z}{d_2}\right) \\ & - \frac{A_2}{2} \cdot e^{-z/d_3} \cdot \sin\left(\frac{364 \cdot 2\pi t}{8760} - \varphi_2 + \varphi_3 - \frac{z}{d_3}\right) - \frac{A_2}{2} \cdot e^{-z/d_4} \cdot \sin\left(\frac{366 \cdot 2\pi t}{8760} + \varphi_2 + \varphi_3 - \frac{z}{d_4}\right), \end{aligned} \tag{12}$$

in which d_1, d_2, d_3, and d_4 are abbreviation constants associated with the diffusivity D and time period of both considered fluctuations. Making use of (7) these constants are calculated by:

$$d_1 = \sqrt{\frac{D \cdot 8760}{\pi}}, \quad d_2 = \sqrt{\frac{D \cdot 24}{\pi}}, \quad d_3 = \sqrt{\frac{D \cdot 8760}{364\pi}}, \quad d_4 = \sqrt{\frac{D \cdot 8760}{366\pi}}. \tag{13}$$

In the present study the analytical model of Eq. (12) was applied for the soil temperature estimation using soil depth and time as independent variables. The parameters D, A_i, and φ, ($i = 1, 2, 3$), were obtained through non-linear regression; the Levenberg–Marquardt algorithm to minimize the loss function, namely the sum of squared residuals, was used as it is implemented in SPSS™ v.10 statistical software. The average temperature, T_a, is considered a model parameter, A_0, obtained through regression.

3. The semi-empirical models

The analytical model of Eq. (12) suggests that T_a is the long-term mean soil surface temperature, constant in the sense that it is achieved after an infinite amount of time. Therefore, our analytical model can be used only in a diagnostic sense to provide soil temperature profiles representing a long term average situation. Diagnostic models have a rather limited area of application, so in order to enhance its applicability we had to render our model with some predictive capabilities. To that end we replaced the constant T_a by a variable, thus obtaining a semi-empirical model based on (12). Another key aspect of model usability is the data requirements, so these should also be kept to a minimum. Daily average soil temperature calculated from all the available depths, T_g, is an obvious selection for a variable T_a. It represents the best short-term approximation of the original assumption of the analytical model, representing a temperature around which the temperature at any depth fluctuates. Even though this average soil temperature presents very good potential in the context of adding predictive skills to our model, it is not a widely available parameter, and this lead us to examine the daily average air temperature at 1.5 m, T_{air}, as an alternative.

4. Field measurements and data

Field measurements of soil temperatures at the surface and at depths of 2, 5, 10, 20, and 30 cm, of a bare soil were obtained from an experimental plot located in the Agricultural University of Athens (AUA) campus, Athens Greece (37°59′7″N and 23°42′24″E). The soil composition was homogenous over an area exceeding 100 m^2 around the thermometers, while the depth of the water table was greater than 18 m, so there was no water table effect on the deep ground temperatures (Krarti et al., 1988). Meteorological data (precipitation, solar and net radiation, air temperature and relative humidity at 2 levels, wind speed and direction at 2 levels, and sunshine duration) were monitored in situ throughout the examined period. The examined period is the 2-year period from April 2002 to March 2004 and all data were recorded on an hourly basis. Overall the meteorological parameters presented an annual variation consistent to the known climatic characteristics of Athens. The rainy period is well defined with only a couple of rainfalls during summer. Some heavy precipitation events, exceeding 30 mm/day, were recorded during the winter months, while April and May were somewhat wetter than usual. The total annual precipitation amount is 324.2 mm. Daily mean air temperature, at 3 m, ranged from –3.1 °C to 32.4 °C, with an average value of 18.9 °C, and solar radiation reached 7.91 kWh m^{-2} d^{-1} during July. Annual average wind speed, at 10 m, was 2.1 m s^{-1} and maximum daily mean wind speed was 5.5 m s^{-1}.

The data from the period April 2003–March 2004 were used for the development of the models while those from the period April 2002–March 2003 were used as an independent data set for the validation of the models. The fact that correct measurement of the soil surface temperature, at an open area lot, requires frequent examination of the probe to ensure that it is not directly exposed to the sun along with our desire to explore the possibility of reducing data requirements led to the construction of the following three soil temperature data sets: the first data set includes all available depths (henceforth referred to as 'ALL'); the second includes all depths except surface (henceforth 'No0'); and the third consists only of soil temperatures at the 2 cm and 30 cm soil depths and will be referred to as '0230'. Both analytical and semi-empirical models were fitted and validated against all three soil temperature data sets.

5. Model performance criteria

Model evaluation was based on some widely used goodness of fit statistics. Namely, Mean Bias Error (MBE, °C), Root Mean Square Error relative to the observed mean value, (RMSE2, %), and the Akaike's Information Criterion (AIC):

$$\mathrm{MBE} = \sum_{i=1}^{N}(P_i - O_i)/N,$$

$$\mathrm{RMSE2} = 100 \cdot \frac{\sqrt{\sum_{i=1}^{N}(P_i - O_i)^2/N}}{\overline{O}}, \tag{14}$$

$$\mathrm{AIC} = N \cdot \ln\left(\sum_{i=1}^{N}(P_i - O_i)^2/N\right) + 2k,$$

where N = number of data points, O_i = observed values, P_i = simulated values, $\overline{O}$ = observed mean, and k the number of model parameters plus one; in our case, $k = 9$ for the analytical model and $k = 8$ for the semi-empirical ones.

6. Results and discussion

Hourly soil temperature at various depths were simulated by the analytical (An) and the two semi-empirical models, one of which is based on the daily average of the soil temperature profile, T_g, and the other is based on the daily average air temperature, T_{air}. Model parameters were fitted using the three different soil temperature data combinations mentioned above, i.e., the 'ALL' data set (all available depths), the 'No0' data set (all depths except surface), and the '0230'data set (for depths at 2 cm and 30 cm). Results are presented for the data from the period April 2002–March 2003 which were used for the validation of the models. As noted previously, for the development of the models data from the period April 2003–March 2004 were used.

Table 1 presents the parameters of the fitted analytical and the semi-empirical models. All models exhibit high correlation coefficients, with the semi-empirical model based on T_g being slightly better in that respect. Note that parameter A_0 is calculated only for the analytical model. The value obtained with the data set ALL, which includes also the soil surface temperature, is 0.3 °C higher than the annual average soil profile temperature (19.7 °C), while the A_0 values

Table 1
Analytical and semi-empirical model parameters fitted using the ALL, No0 and 0230 data sets

	ALL			No0			0230		
	An	T_g	T_{air}	An	T_g	T_{air}	An	T_g	T_{air}
r2	0.90	0.95	0.91	0.94	0.97	0.93	0.93	0.96	0.93
A0	19.96 (0.01)			19.8 (0.01)			19.71 (0.02)		
A1	13.03 (0.02)	0.1 (0.02)	2.36 (0.02)	12.82 (0.02)	0.02 (0.01)	2.25 (0.02)	12.85 (0.03)	−0.23 (0.02)	3.14 (0.04)
A2	4.61 (0.05)	4.82 (0.04)	4.87 (0.05)	3.65 (0.05)	3.64 (0.03)	3.69 (0.05)	3.62 (0.06)	3.58 (0.05)	10 (0.4)
A3	1.8 (0.02)	1.77 (0.01)	1.77 (0.02)	1.7 (0.02)	1.67 (0.01)	1.68 (0.02)	1.69 (0.03)	1.67 (0.02)	1.68 (0.03)
F1	4.337 (0.002)	4.2 (0.1)	4.26 (0.009)	4.328 (0.001)	6.6 (0.6)	4.261 (0.008)	4.33 (0.002)	5.15 (0.09)	4.5 (0.01)
F2	12.49 (0.01)	6.215 (0.008)	6.22 (0.01)	6.33 (0.01)	12.638 (0.008)	0.07 (0.01)	6.31 (0.02)	12.61 (0.01)	6.32 (0.02)
F3	4.063 (0.004)	4.091 (0.003)	4.099 (0.004)	3.815 (0.006)	3.799 (0.004)	3.817 (0.006)	3.782 (0.007)	3.76 (0.006)	4.8 (0.03)
D ($\times 10^3$)	0.75 (0.01)	0.595 (0.007)	0.556 (0.008)	1.46 (0.02)	1.59 (0.02)	1.45 (0.03)	1.26 (0.04)	1.56 (0.06)	0.035 (0.002)

Values in parentheses are the parameters' standard errors.

obtained excluding the soil surface temperature differ at most 0.1 °C from the overall annual mean. Amplitudes A_1 (annual soil temperature fluctuation) have larger values in the analytical model than in the semi-empirical ones, since the added temperature-related independent variable already contains much of the information pertinent to the annual cycle. On the other hand, since the extra independent variable is a daily averaged value, diurnal amplitudes A_2 and A_3 are not systematically differentiated. Diffusivity D, in $10^3\ m^2\ h^{-1}$, ranges between 0.75 and 0.56 when the ALL data set is considered. When surface temperature is not included, the models provide consistent diffusivities ranging between 1.26 and 1.59, with the exception of the semi-empirical model based on air temperature and the "0230" data set that gives a diffusivity of $0.035 \times 10^3\ m^2\ h^{-1}$.

The annual variation of the observed and simulated temperatures at the surface (0 cm), using the data set ALL, is presented in Fig. 1, while Figs. 2 and 3 present the respective results at the depths of 2 cm and 20 cm. The observed hourly soil temperature is fairly well approximated by both the analytical and the semi-empirical models. All models seem to capture the main characteristics of the annual course of the variation of soil temperature, with maximum during summer and minimum during winter. The semi-empirical models, provide a quite realistic annual variation (Fig. 1b and c). The maximum diurnal variability of surface temperature is correctly predicted to occur at late summer, but with smaller amplitude since all models predict, to roughly the same degree, significantly lower surface temperature maxima. The T_g semi-empirical model gives a fairly good prediction of the diurnal minimum of the soil surface temperature, while the analytical model under-predicts and the T_{air} semi-empirical model over-predicts this parameter (Fig. 1). The degree of underestimation of the diurnal maximum temperature diminishes as we move to greater depths, where the influence of the external factors driving the soil temperature variability tends to smooth out (Figs. 2 and 3).

The results obtained from the semi-empirical models, as already mentioned, fluctuate in a much more realistic way than those of the analytical model. This, of course is due to the propagation of the variability of the parameters used to drive the models. Thus, the model obtained using the daily average soil temperature profile, T_g, as driving parameter, provides a variation very similar to the actual one, especially below the surface (Figs. 2b and 3b). The model predicts slightly exaggerated minima during late winter – early spring affected by the respective intense response of the soil surface temperature that is transferred into the daily T_g values (Fig 3b). This propagation of variation is more clearly observed in the semi-empirical model based on daily air temperature, T_{air}, where during late autumn and winter, even though the day-by-day variation is plausible, the diurnal variation is minimized reflecting the minimal diurnal variation of air temperature during that period (Figs. 1c, 2c, 3c).

The goodness of fit statistics, presented in Table 2, suggest that, on average, the semi-empirical models underestimate the hourly soil surface temperature, while the analytical model slightly overestimates it. The respective MBEs of the analytical model is in the order of +0.4 K, while the semi-empirical model based on T_g underestimates soil surface temperature by 0.4 K and the air temperature model by a little more than 1.5 K. Deeper into the ground, the T_{air} model underestimates soil temperature at any depth by approximately 1°. The analytical and the T_g semi-empirical models provide soil temperatures that, on

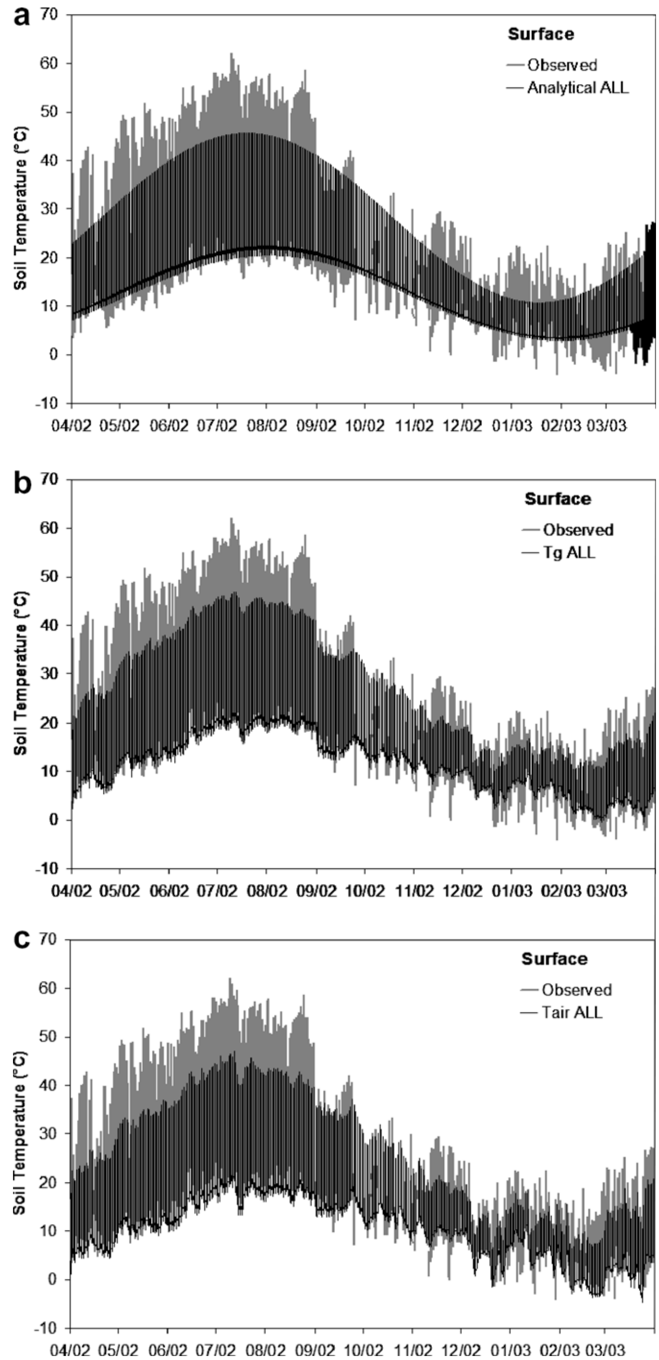

Fig. 1. Comparison of observed hourly soil surface temperature values for the period 01/04/2002 31/03/2003 with predictions made by the: (a) analytical model, (b) T_g model, and (c) T_{air} model, fitted with the ALL data set.

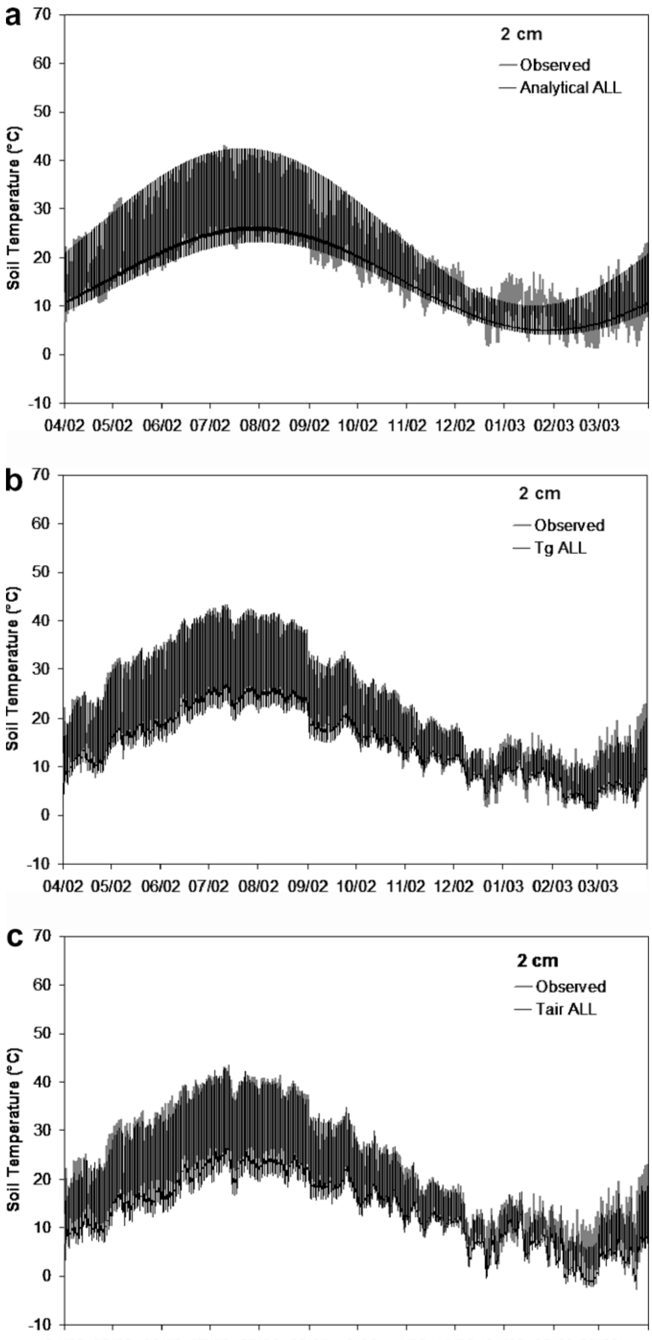

Fig. 2. Comparison of hourly soil temperature values at 2 cm, for the period 01/04/2002 31/03/2003 with predictions made by the: (a) analytical model, (b) T_g model, and (c) T_{air} model, fitted with the ALL data set.

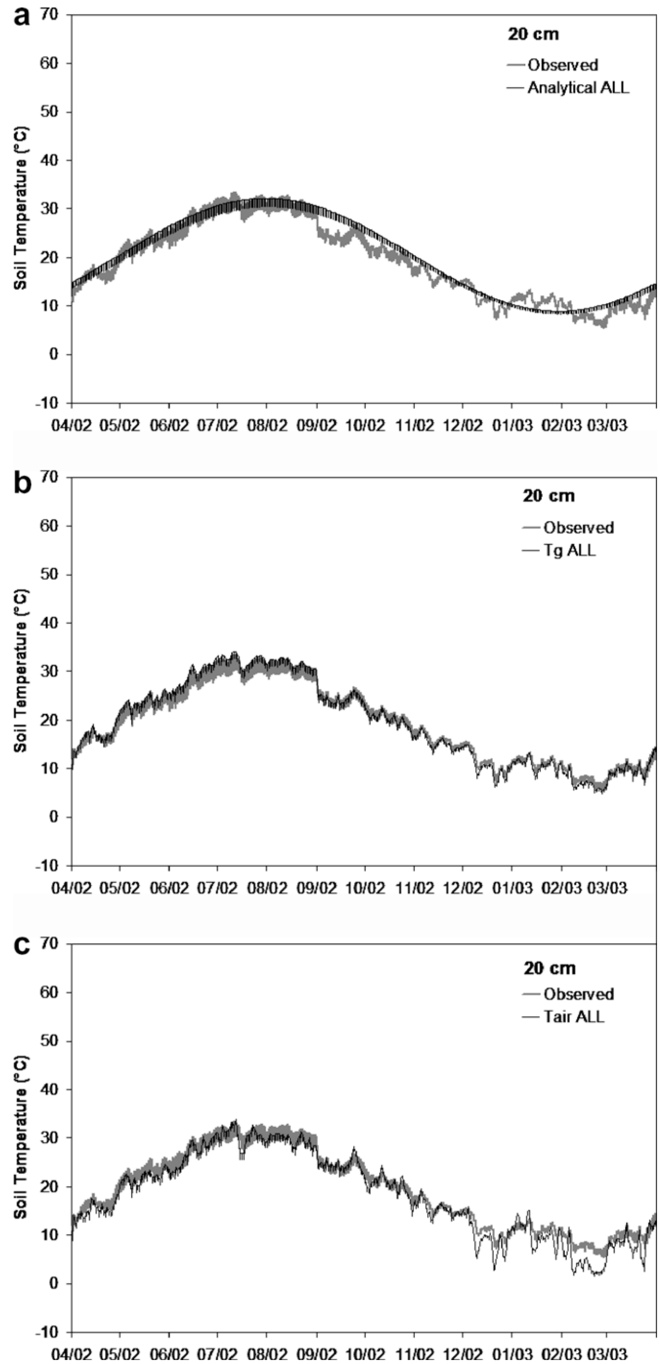

Fig. 3. Comparison of hourly soil temperature values at 20 cm, for the period 01/04/2002 31/03/2003 with predictions made by the: (a) analytical model, (b) T_g model, and (c) T_{air} model, fitted with the ALL data set.

average, are either less than one degree higher or equal to the actual ones. The worst performance in terms of MBE is that of the semi-empirical model based on T_{air}, which yields a consistent underestimation of the soil temperature profile by 1.2 K. The highest RMSE2 values are always assigned to soil surface temperature predictions. Beyond that, the analytical model presents diminishing RMSE2 values as we move deeper, while the semi-empirical models present a U shaped RMSE2 profile with depth, the minimum RMSE2 being observed at 10 cm. Overall, the models fitted with the ALL data set present, as expected, RMSE2 lower for the surface temperatures, worse for the temperature at 2 cm, and practically the same as the rest of the models for the other depths. T_g based model presents the lower RMSE2, 12–14%, followed by the analytical and the T_{air} models with RMSE2 values between 15% and 17%. The T_{air} model based on the “0230”data set has the worst RMSE2 value mainly due to its failure to reproduce the surface temperatures. Noting that this model has also been singled out due to its very low diffusivity, it seems that

Table 2
Annual goodness of fit statistics for the analytical and semi-empirical models of hourly soil temperature at various depths

	MBE					RMSE2					AIC				
	0 cm	2 cm	10 cm	30 cm	All	0 cm	2 cm	10 cm	30 cm	All	0 cm	2 cm	10 cm	30cm	All
Analytical															
ALL	0.5	1.1	0.9	1.0	0.9	27	18	12	9	16	26161	21690	14751	10218	116873
No0	0.3	0.9	0.8	0.8	0.7	32	16	12	9	17	28746	19061	14373	9723	121541
0230	0.2	0.8	0.7	0.7	0.7	32	15	12	9	17	28962	18755	14097	9120	121639
Semi-empirical T_g															
ALL	−0.4	0.2	0.0	0.1	0.0	24	13	5	9	12	24117	15192	−1743	8989	86534
No0	−0.4	0.2	0.0	0.1	0.0	30	9	4	9	14	27698	9666	−3063	8492	97802
0230	−0.4	0.2	0.0	0.1	0.0	31	9	4	8	14	28184	9660	−3625	7225	99663
Semi-empirical T_{air}															
ALL	−1.7	−1.0	−1.2	−1.1	−1.2	26	15	11	12	15	25512	18326	12405	14550	109828
No0	−1.7	−1.0	−1.2	−1.1	−1.2	31	12	10	12	16	28470	14913	11864	14728	116863
0230	−1.6	−1.0	−1.2	−1.1	−1.2	49	12	14	11	23	35691	14241	16636	13347	154379

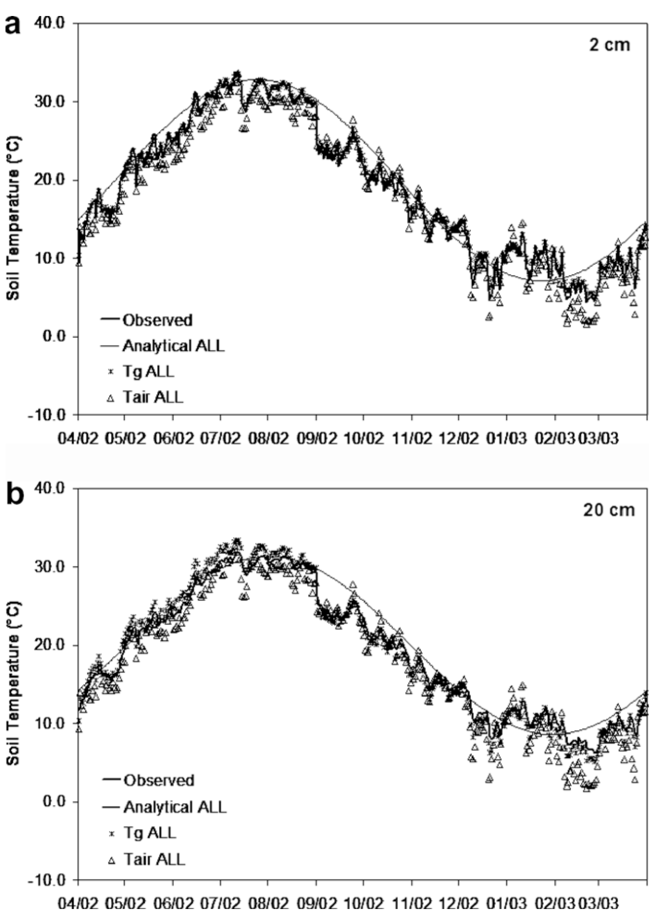

Fig. 4. Comparison of observed mean daily soil temperature values for the period 01/04/2002 to 31/03/2003, to those modeled by the analytical and semi-empirical models fitted with the ALL data set, at (a) 2 cm, and (b) 20 cm depth.

using soil temperatures only from two depths is not enough to provide a meaningful model.

The improvement of the analytical model's statistics with depth is the result of the diminishing influence of the external driving factors that are not explicitly included in the model. The U-shaped performance curve of the semi-empirical models, on the other hand, is a result of the intense differentiation of the observed soil temperature variability at the various depths. The external variables (T_g and T_{air}) are daily average values thus they can only contribute to the annual variability of the predicted soil temperatures. When it comes to diurnal variability the models have to rely on the parameters A_2 and A_3 which, being the result of a regression procedure, are bound to reproduce the variation of the soil temperature at a mean depth in a better way than the variation at depths close to the margins of the domain considered. When data from all depths are considered, and keeping in mind that the attenuation of temperature variability with depth is not linear, this so-called mean depth should be shifted towards the surface, thus making the 10 cm depth a valid candidate for improved model performance (Table 2). The AIC indicates that, within each modeling approach, the models marked as ALL perform better. This of course was to be expected since these models are developed using the soil surface temperatures along with the rest, thus implicitly accounting for the soil surface properties. The overall best model is the one built using all the available data and T_g as an independent variable along with depth and time. The rest of the T_g models rank second and third, followed by the T_{air} ALL and "No0" models. Apparently, T_{air} models are performing worse than T_g models because the later present a variation that is smoother and on average closer to the 'theoretical' long term average value of the soil surface temperature.

Finally, it is interesting to evaluate model predictions on a daily basis. The annual course of the observed daily soil temperature values, along with the estimated respective values (daily average) of the analytical and the semi-empirical models, fitted with the ALL data set, are presented in Fig. 4. Also, the calculated annual and seasonal goodness of fit statistics derived from the daily averaged hourly estimations of the analytical and semi-empirical models, fitted with the ALL data set, are shown in Table 3. The T_{air} semi-empirical model consistently underestimates soil temperature at any depth with the exception of upper soil temperatures (2 and 5 cm) during autumn. The T_g semi-empirical model, 'on the other hand' has a mixed behaviour. During

Table 3
Annual and seasonal goodness of fit statistics for daily soil temperature at various depths, estimated by the analytical and semi-empirical models

	MBE (°C)				RMSE2 (%)				AIC			
	2 cm	5 cm	10 cm	20 cm	2 cm	5 cm	10 cm	20 cm	2 cm	5 cm	10 cm	20 cm
Annual												
Analytical ALL	1.1	1.0	0.9	0.9	14	13	12	10	701	662	590	492
T_g ALL	0.2	0.1	0.0	0.0	3	2	2	5	−463	−724	−848	−1
T_{air} ALL	−1.0	−1.1	−1.2	−1.2	9	9	10	11	382	407	454	538
Spring												
Analytical ALL	1.7	1.5	1.3	1.1	15	13	12	10	181	165	139	107
T_g ALL	−0.1	−0.1	0.0	0.3	2	2	2	6	−150	−209	−229	−6
T_{air} ALL	−1.5	−1.5	−1.5	−1.2	11	11	11	11	121	126	127	126
Summer												
Analytical ALL	0.8	0.6	0.5	0.3	5	4	4	3	85	64	34	0
T_g ALL	0.0	0.0	0.4	1.1	1	1	1	4	−199	−242	−143	45
T_{air} ALL	−1.7	−1.7	−1.4	−0.9	6	6	6	5	135	135	119	86
Autumn												
Analytical ALL	2.3	2.3	2.3	2.3	31	30	28	25	228	223	213	2.3
T_g ALL	0.4	0.2	−0.1	−0.6	6	4	2	7	−69	−133	−243	0.4
T_{air} ALL	0.4	0.2	−0.1	−0.4	8	7	8	11	−30	−40	−19	0.4
Winter												
Analytical ALL	−0.4	−0.4	−0.4	−0.1	59	55	47	35	206	197	176	137
T_g ALL	0.4	0.2	−0.2	−0.9	14	9	5	19	−60	−135	−224	27
T_{air} ALL	−1.3	−1.4	−1.8	−2.3	39	42	49	58	129	145	181	223

autumn and winter it overestimates daily upper soil temperatures and underestimates deeper soil temperatures (10 and 20 cm). This pattern is reversed and significantly weaker during spring, while during summer the model overestimates the deeper soil temperatures only. Finally, the analytical model underestimates only during winter and overestimates during the rest of the year especially during the transient seasons.

Examining the seasonal statistics, it seems that winter is more difficult to model since the respective RMSE2 values are much larger than the rest of the seasons, with summer having the lowest values. The dependence of RMSE2 on the observed average soil temperatures during the two seasons, ranging from 9.6 °C in winter to 31.5 °C in summer, would account for a threefold RMSE2 during winter as compared to the summer value, yet the difference observed here is much more than five fold, supporting the inference of much larger differences between observed and modeled values. The analytical model is the one least favored by the AIC. The semi-empirical model based on T_g should be selected as the best one in terms of overall seasonal performance. The semi-empirical model based on T_{air}, is far less successful in estimating daily soil temperatures, yet it should not be discarded, since requiring only a widely available meteorological parameter it provides quite useful results, with RMSE2 less than 11%, except for winter, when it produces abnormally low temperatures.

7. Conclusions

According to the analysis presented above it has been demonstrated that simple regression models can be used to estimate and predict the soil temperature profile with minimal data requirements. The analytical model does not require any data at all to estimate the soil temperature profile, while the semi-analytical models require only average daily values of soil or air temperature. Based on the above the following conclusions can be drawn:

– A very good correlation between the observational and the model-estimated soil temperature patterns at the various depths has been achieved.
– The model estimated soil temperature values compared fairly well with the measured ones with acceptable statistical errors in almost all cases.
– It appears reasonable to suggest that the developed models can serve as useful tools for estimating soil temperature at various depths in the context of various environmental and energy applications.

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